Overview of Probabilistic Modeling of Woven Ceramic Matrix Composites

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ABSTRACT

An overview of current research efforts in understanding the cause of variability in the thermo-mechanical properties of woven ceramic matrix composites is presented. Statistical data describing the architectural variability from micrographs is used to generate artificial ceramic matrix composite specimens. A large number of artificial specimens can then be analyzed with finite element analysis and analytical methods to determine which architectural components contribute most to the variability in properties. It was found that variation in tow width, tow spacing, and tow height alone cause some degree of variability in the mechanical properties, but additional architectural variations such as ply shifting and the associated voids must also be considered in order to capture a more complete representation of the variability in mechanical properties.

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INTRODUCTION

Thermal protection systems and hot structures are of current interest for supersonic and hypersonic vehicles. These vehicles must be light weight and are subjected to extreme temperatures (2000-4000°F) and structural loads. Materials that have been used in the past, such as metal alloys, do not satisfy these requirements. As shown in Figure 1, ceramic matrix composites (CMCs) have potential for use in harsh thermo-structural applications [1].

However, ceramic matrix composites do not come without drawbacks. Current CMC material property prediction techniques are based on semi-empirical relationships [2]. If the fiber architecture, volume fractions, or constituent materials change, a new set of tests must be completed, adding to the cost and time needed to implement the materials. Therefore, the goal of ongoing research in this field is to develop physics-based models that are capable of predicting material properties such as thermo-elastic constants and strength with limited to no empirical information.

Preliminary evaluation of some ceramic matrix composites indicates that there is considerable variability in their mechanical properties. Randomness in the tow architecture, as well as the randomly shaped and spaced voids that are produced as a result of the manufacturing process, are major features contributing to this variability [3–6]. The focus of the research discussed in this paper is to relate variability in the composite architecture to variability in the thermo-mechanical properties and strength. When addressing engineering problems related to variability and uncertainty, a large amount of simulations are often necessary. Therefore, it is important to develop efficient methods to propagate the variability in the composite material to variability in its properties. This can be done by applying statistical information from micrographs of the composites to semi-analytical and finite element models. The research may be useful to both material manufacturers and design engineers in that the cause of the variability in thermo-mechanical properties and strength and the implications that the architectural variability will have on its use in design will be understood.

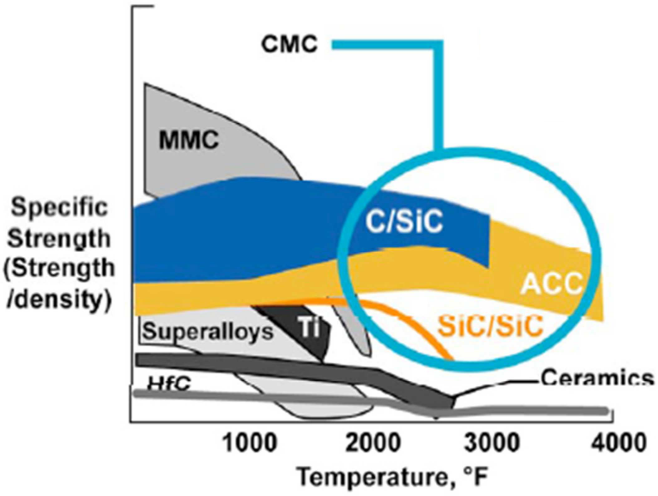

Figure 1: Comparison of current material capabilities [1]

COMPOSITE CHARACTERIZATION

Woven ceramic matrix composites are candidate materials for future supersonic and hypersonic vehicle components due to their strength and fracture toughness at elevated temperatures. For this work, the composite system under investigation is a CVI (Chemical Vapor Infiltration) SiC/SiC eight-ply 5HS (harness satin) weave material. The image in Figure 2 is a 3D representation of the weave for a 5HS unit-cell. The composite has continuous Sylramic-iBN fiber tows (20 ends per inch) woven into a five-harness woven fabric perform in a [0°/90°] pattern. A silicon-doped boron nitride coating is deposited on the surface of the individual fibers in the tows. The fiber preform is then infiltrated with a CVI-SiC matrix which fills the tows and forms a thin matrix coating around the tows [7].

Initial evaluation of the composite system has shown large amounts of variability in the mechanical properties. A 2D image of one cross section of the SiC/SiC composite, obtained by Goldberg, et al. [7] is shown in Figure 3. The black areas in the interior of the cross section represent voids (the black area comprising the border of the image is not voids), which vary in location, size, and shape. Other 2D cross sections are not identical to the one shown, but rather, exhibit different random distributions of the voids and the microstructural characteristics such as tow size, shape, and spacing.

Statistical data of various aspects describing the composite architecture was obtained by Goldberg, et.al [7]. The architectural parameters' statistical data are presented in Table I and include, tow width, tow height, tow spacing, and constituent volume fractions. The parameters were found to be normally distributed. Note that the values in parentheses are standard deviations. Other parameters that describe the unit cell, as labeled in Figure 4, were either estimated from the micrographs or are a function of the other architectural variables, and were held constant.

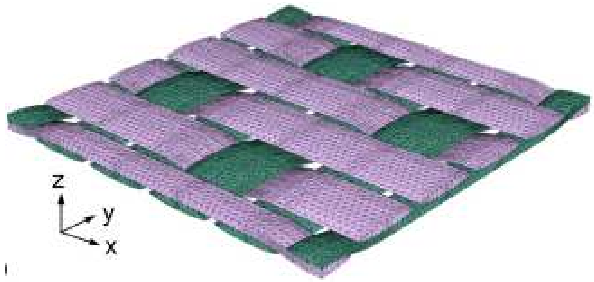

Figure 2: 3D finite element model of 5 harness satin weave [8]

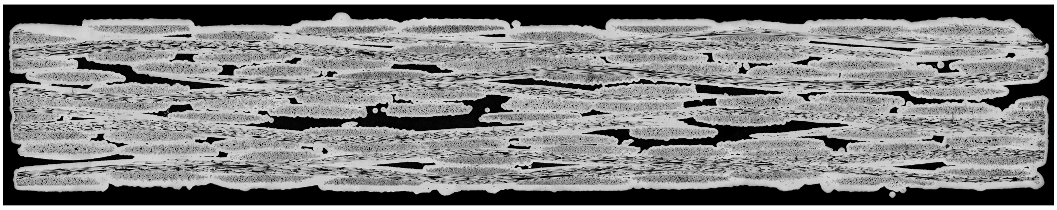

Figure 3: 2D cross section of the SiC/SiC composite microstructure [7]

TABLE I: SUMMARY OF VOLUME FRACTION AND GEOMETRIC CHARACTERISTICS FOR 3 REAL SAMPLE SPECIMENS

	% Void	**% Matrix**	**% Tow**	***w* (mm)**	***s* (mm)**	***h* (mm)**
Specimen 1	3.2	33.8	63.0	1.14 (0.08)	1.27 (0.05)	0.12 (0.01)
Specimen 2	4.8	32.4	62.8	1.15 (0.08)	1.27 (0.06)	0.12 (0.01)
Specimen 3	3.5	32.6	63.9	1.14 (0.08)	1.27 (0.05)	0.12 (0.01)
Mean Value	3.8	32.9	63.2	1.14	1.27	0.12
St. Dev.	0.9	0.8	0.6	0.08	0.05	0.01

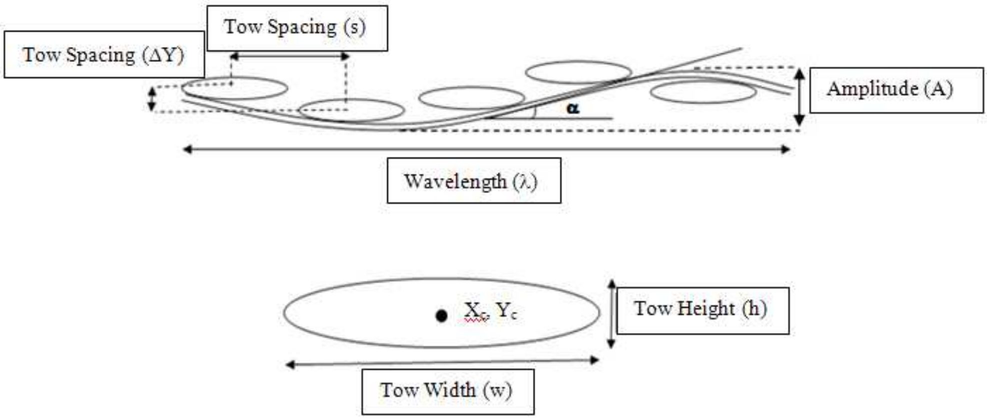

Figure 4: Geometry of unit cell

For this work, the focus was on modeling an RVE of the 8 ply 5 HS composite in order to keep the size of the problem tractable while capturing the important statistical characteristics. Due to the large amount of variability, it is difficult to define an RVE in the traditional manner, in which the RVE is a statistical equivalent representation of the larger specimen. The RVE shown in Figure 5 consists of two unit cells, with one flipped upside down (as is done during material fabrication), and a uniform shifting of one tow offset applied. The effects of neglecting ply shifting will be discussed below.

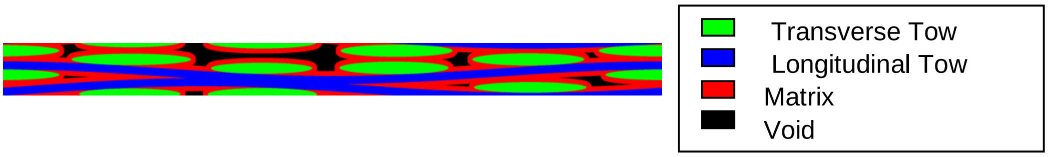

Figure 5: Example of a randomly generated RVE

ANALYSIS METHODS

As previously mentioned, it is important to consider computational cost when applying probabilistic methods. Semi-analytical micromechanics simulations are computationally inexpensive, but have known short-comings for some applications (such as those with large inter-tow voids seen in some woven CMCs). The opposite is true for finite element analysis (FEA). Using response surface techniques, in which varying architectural parameters can be related to thermo-mechanical properties and strength, the number of finite element analyses can be reduced and/or the semi-analytical methods can be adjusted to properly account for voids based on FEA results.

Finite Element Analysis

The RVEs, with varying architectural parameters (tow width, tow height, and tow spacing), were generated as images, which were then meshed with the open source software, OOF2 [11]. The mesh is imported into commercial software, ABAQUS, for FEA [12]. A combination of triangular and quadrilateral plane strain elements was used. A finite element analysis based micromechanics approach with periodic boundary conditions was used to determine the effective elastic moduli, Poisson's ratios and coefficients of thermal expansion (CTEs) of the RVE using the constitutive equation

$$\underset{6\times1}{\{\sigma\}} = \underset{6\times6}{[C]}\left\{\underset{6\times1}{\{\varepsilon\}} - \underset{6\times1}{\{\alpha\}}\Delta T\right\} \tag{1}$$

where the stresses and strains are macroscopic or volume averaged quantities, C is the stiffness matrix, α is the matrix of CTEs and ΔT is the temperature difference measured from the reference temperature.

Selective Averaging Method (SAM)

The Selective Averaging Method is an analytical method for determining elastic constants that was proposed by Sankar and Marrey [9]. The method simply requires a RVE and knowledge of the constituent material properties and their location. For this approach, the RVE is divided into slices. The slices are subdivided into elements. Then, the elastic constants of the constituents are averaged by selecting either the isostress or isostrain condition, depending on which is the more appropriate assumption for the given level being accounted for. For example, iso-strain is assumed when moving from the element level to the slice. This implies that every element within the slice has the same amount of strain. However, each slice can have a different strain. Then, from the slice to macroscale level, there is an isostress assumption in which every slice has an equivalent average stress. This method was shown to agree very well with FEA results for the 5HS weave that was used for comparison in their study.

Response Surfaces

When it is desired to determine the response at a large number of data points, it is typical to perform analyses at a small set of data points, which are then fit with a polynomial response surface. The number of simulations needed is dependent on the number of variables, as well as the degree of the polynomial of your desired fit. Since all of the polynomials used in this work are linear, $2(n+1)$ analyses are necessary, where n is the number of variables. The relationship between the variables and thermo-mechanical properties is then given by

$$response = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n + c_{n+1} \tag{2}$$

where c_n is the coefficient and x_n is the variable. The coefficients of the response surface provide insight into how sensitive the response is to a given variable. The response surface can also be used to generate additional results based on interpolation to determine the statistical distribution of the properties (Normal, Weibull, etc).

RESULTS AND DISCUSSION

Results from the finite element analysis and response surface results are presented in Table II. It is known that voids have a more detrimental effect on the out of plane moduli than the in-plane moduli, which is observed in the results below [10]. Huang and Talreja [10] also observed that voids have the most impact on the out-of-plane shear modulus (G_{13}). From the response surface coefficients, it was found that tow width, tow spacing, and volume fractions had the most significant impact on variability in the mechanical properties. Similar results were generated using the semi-analytical method SAM. It was found that SAM agrees well with the below results for in-plane moduli. However, the out-of-plane modulus is over predicted by approximately two times the FEA and approximate experimental results. The same is true for other analytical methods as well as finite element models that do not explicitly model the inter-tow voids [8].

The inter-tow voids do not only affect the mean value of mechanical properties, but they have also been found to affect the amount of variability exhibited by the mechanical properties. The results presented in Table II indicate that the variability in the through thickness modulus is approximately 5% of the mean. However, finite element analyses of full specimens like that shown in Figure 3 indicated that the variability is much higher (on the order of 20% or more) [7]. As previously mentioned, variable ply shifting was neglected in the current models. Preliminary efforts have indicated that variable ply shifting is important to consider, given the magnitude of the affect it has on the voids size, shape, and spacing which inherently affects the mechanical properties and strength. A comparison of the effect of a few different tow offsets (and thus different void configurations) on different artificial specimens indicates that the shifting significantly affects both the mean and the amount of variability, as shown in Figure 6.

Work in progress is focused on determining if the voids can be characterized in such a way that the resulting mechanical properties and strength can be predicted based on that information. The response surface results presently used are based on only one void configuration as a result of one tow offset in ply shifting. If the shifting pattern is altered, it is likely that the response surface would have to be altered also. The information about how voids affect mechanical properties and strength, then, would be valuable for finite element results, and may provide insight into how analytical models can be adjusted if voids are an issue in the given composite system.

Table II: SUMMARY OF FEA AND RESPONSE SURFACE RESULTS

	Finite Element Analysis Results		Response Surface Results	
	Mean	Standard Deviation	Mean	Standard Deviation
E_1 (GPa)	231.0	5.0	230.4	3.6
E_2 (GPa)	259.9	1.9	260.0	1.9
E_3 (GPa)	105.8	6.2	106.2	4.4
ν_{12}	0.174	0.005	0.174	0.003
ν_{13}	0.202	0.004	0.201	0.003
ν_{23}	0.123	0.006	0.123	0.005
G_{12} (GPa)	74.5	5.2	74.1	3.1
G_{13} (GPa)	20.6	3.6	20.4	2.3
G_{23} (GPa)	44.8	1.7	44.9	0.9
α_1 (10^{-6}/°C)	4.65	0.001	---	---
α_2 (10^{-6}/°C)	4.65	0.001	---	---
α_3 (10^{-6}/°C)	4.62	0.001	---	---

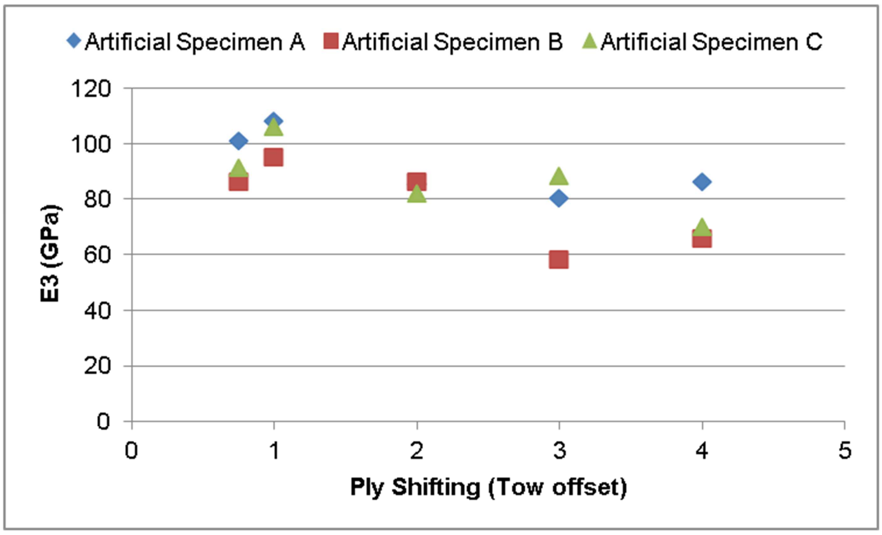

Figure 6: Effect of ply shifting on the out-of-plane modulus

CONCLUSIONS

Progress has been made in determining how various architectural parameters of 5H SiC/SiC composites affect their thermo-mechanical properties. It was found that the architectural parameters considered in this work (tow spacing, tow height, and tow width) do not capture the full extent of the variability exhibited by the specimens, but do play a minor role. The method of employing statistical information from the micrographs for use in artificial models works well, and will be useful in determining the specific effects of voids on the thermo-mechanical properties and strength using finite element or analytical micromechanics.

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REFERENCES

[1] D. E. Glass, “Ceramic Matrix Composite (CMC) Thermal Protection Systems (TPS) and Hot Structures for Hypersonic Vehicles 1,” in *15th AIAA Space Planes and Hypersonic Systems and Technologies Conference*, 2008, no. March 2007.

[2] J. Z. Gyekenyesi, P. L. N. Murthy, and S. K. Mital, “Life Prediction for a CMC Component Using the NASALIFE Computer Code,” vol. 213887, 2005.

[3] L. N. Murthy, K. Mital, and R. Shah, “Probabilistic Micromechanics Macromechanics for Ceramic Matrix Composites,” *NASA Technical Memorandum*, no. 4766, 1997.

[4] A. R. Shah, P. L. N. Murthy, S. K. Mital, and R. T. Bhatt, “Probabilistic Modeling of Ceramic Matrix Composite Strength,” *Journal of Composite Materials*, vol. 34, no. 8, pp. 670-688, Apr. 2000.

[5] J. A. Dicarlo, H. M. Yun, G. N. Morscher, and R. T. Bhatt, “SiC / SiC Composites for 1200 ° C and Above,” *NASA Technical Memorandum*, no. 213048, 2004.

[6] F. E. Heredia, M. Y. He, and A. G. Evans, “Mechanical performance of ceramic matrix composite I-beams,” *Composites Part A: Applied Science and Manufacturing*, vol. 27A, pp. 1157-1167, 1996.

[7] R. K. Goldberg, P. J. Bonacuse, and S. K. Mital, “Investigation of Effect of Material Architecture on the Elastic Response of a Woven Ceramic Matrix Composite,” *NASA Technical Memorandum*, vol. 215806, 2012.

[8] S. K. Mital, B. A. Bednarcyk, S. M. Arnold, and J. Lang, “Modeling of Melt-Infiltrated SiC / SiC Composite Properties October 2009,” *NASA Technical Memorandum*, vol. 215806, 2009.

[9] B. V. Sankar and R. V. Marrey, “Analytical method for micromechanics of textile composites,” *Composites Science and Technology*, vol. 57, no. 6, pp. 703-713, Jan. 1997.

[10] H. Huang and R. Talreja, “Effects of void geometry on elastic properties of unidirectional fiber reinforced composites,” *Composites Science and Technology*, vol. 65, no. 13, pp. 1964-1981, Oct. 2005.